Mechanics

Position, starting with 2 dimensions

We can measure our position forward with positive numbers and backwards with negative numbers. The bike is at 0.

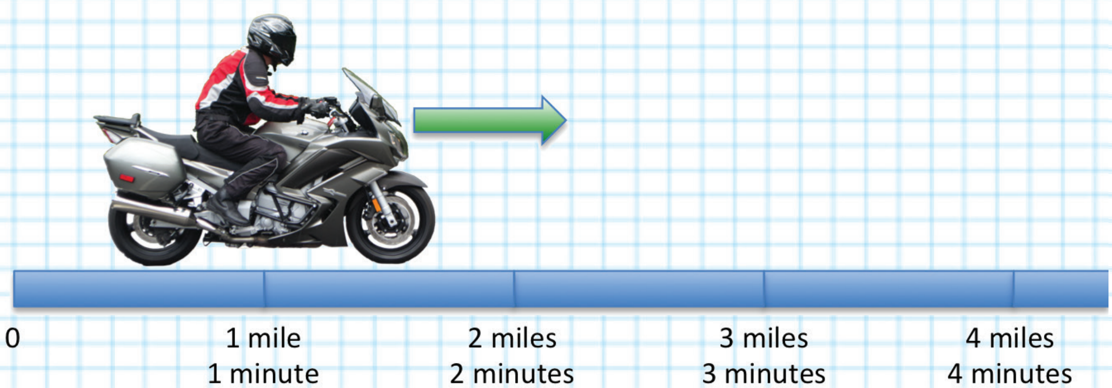
Speed (Average)

If we take two odometer readings, one at the start of the hour and another two hours later, our average speed is:

0	0	1	0	0	0
0	0	1	0	9	0

$$\text{Speed} = \frac{(1090 - 1000)}{2 \text{ hours}} = 45 \text{ mph}$$

Speed



If after 1 minute the bike has traveled 1 mile, we can say:

$$\text{Speed} = \frac{1 \text{ mile}}{1 \text{ minute}} = 1 \text{ mile/min}$$

Or, if we prefer a more standard measure:

$$\text{Speed} = \frac{1 \text{ mile}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 60 \text{ mph}$$

Velocity

Speed is a scalar quantity (it only has magnitude, no direction.)

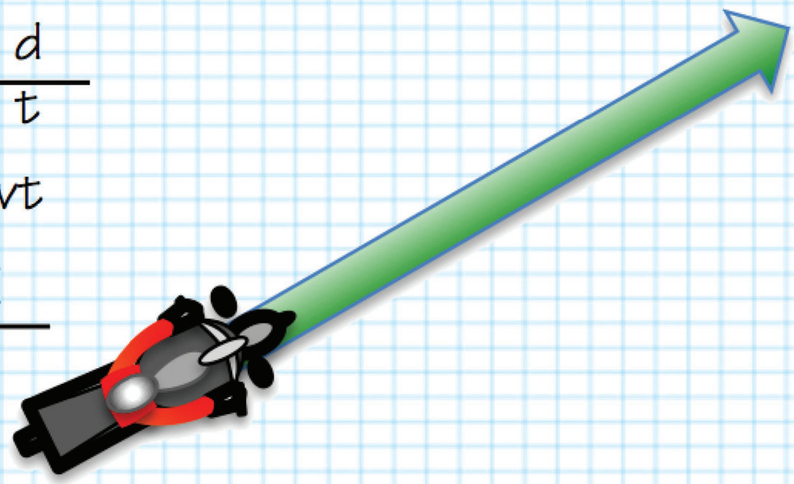
Velocity is a vector quantity (it has magnitude and direction).

Direction is like angles on a grid, or on a compass. The bike is heading 30° north of east, or in pilot-speak: heading 060°

$$\text{Velocity } v = \frac{d}{t}$$

$$\text{Distance } d = vt$$

$$\text{Time } t = \frac{d}{v}$$



Consistent Units

We should keep the units consistent. Anything can be multiplied by one without changing the answer, so multiplying by a unit conversion keeps the answer true while making it more useful. 1 hour equals 60 minutes, for example. 1 hour divided by 60 minutes, therefore, equals 1. So multiplying something by 1 hour and then dividing by 60 minutes keeps the answer correct. In the example, 50 mph = 84 fps.

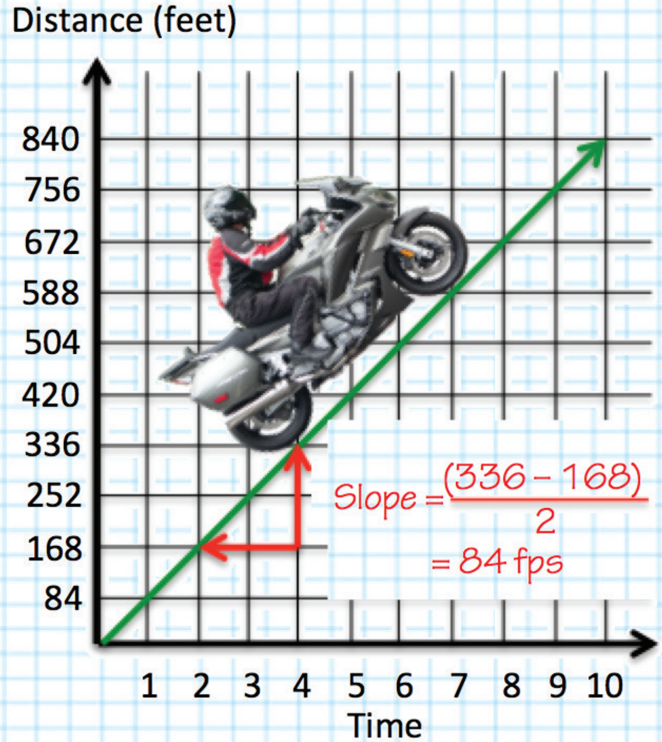
$$\left[\frac{50 \text{ miles}}{1 \text{ hour}}\right] \times \left[\frac{1 \text{ hour}}{60 \text{ mins}}\right] \times \left[\frac{1 \text{ minute}}{60 \text{ secs}}\right] \times \left[\frac{6076 \text{ feet}}{1 \text{ mile}}\right] = \left[\frac{84 \text{ feet}}{1 \text{ second}}\right]$$

$$\left[\frac{50 \cancel{\text{miles}}}{1 \cancel{\text{hour}}}\right] \times \left[\frac{1 \cancel{\text{hour}}}{60 \cancel{\text{minutes}}}\right] \times \left[\frac{1 \cancel{\text{minute}}}{60 \text{ seconds}}\right] \times \left[\frac{6076 \text{ feet}}{1 \cancel{\text{mile}}}\right] = 84 \text{ fps}$$

Some of the units cancel each other out. A "minute" divided by another "minute" equals 1 and disappears from the answer. In our example, every unit except feet and seconds disappear, yielding feet per second.

Velocity on a Graph

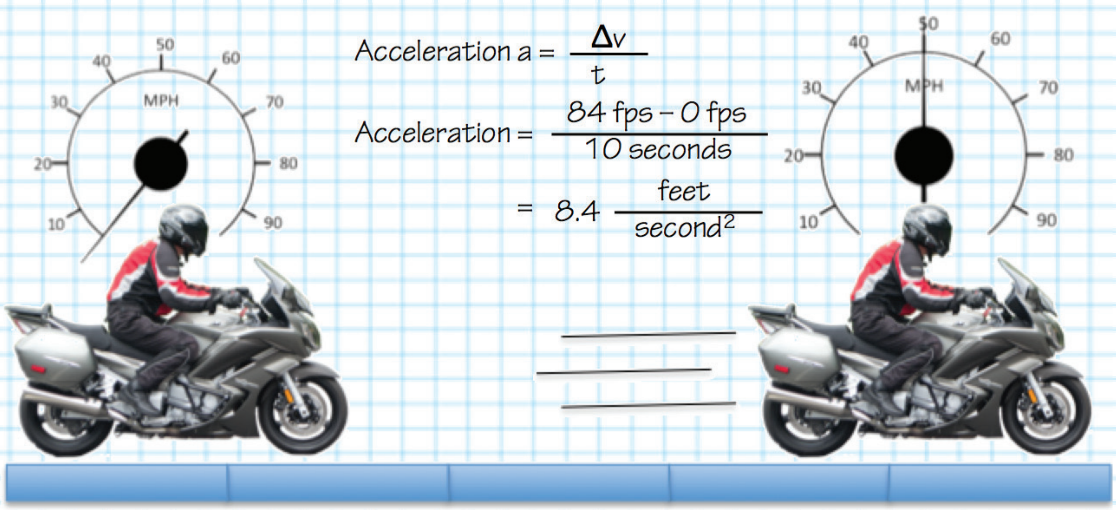
You can also plot distance versus time to come up with a graphical representation of velocity. The slope of the distance change along any point of the line will show the velocity over that time span. Since our velocity is constant — neither accelerating or decelerating — any two points will yield a velocity of 84 fps



Acceleration

Change in velocity per unit of time

Δ symbol signifies change in values



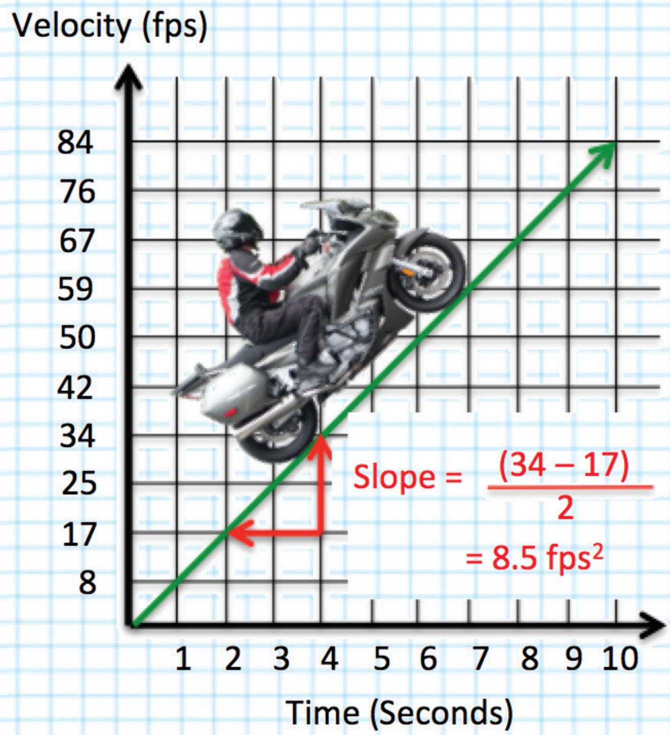
$$\text{Acceleration } a = \frac{\Delta v}{t}$$

$$\text{Acceleration} = \frac{84 \text{ fps} - 0 \text{ fps}}{10 \text{ seconds}}$$

$$= 8.4 \ \frac{\text{feet}}{\text{second}^2}$$

Acceleration on a Graph

The slope of the velocity change along any point of the line will show the acceleration over that time span. Since our acceleration is constant any two points will yield an acceleration of 8.5 fps^2.



$$\text{Acceleration } a = \frac{\Delta v}{t}$$

Where $\Delta v = v_1 - v_0$

Then: $v = at$

If $v_0 = 0$ (starting velocity) and

$v_1 = at$ (ending velocity)

Average velocity, $v_{average} = \frac{1}{2}at$

Since $d = vt$ that means $d = (\frac{1}{2}at)t$ and that means:

Distance, given constant acceleration: $d = \frac{1}{2}at^2$

Jerk

"Jerkiness" can be measured as the second derivative of position. If velocity is constant, acceleration is zero, if acceleration is constant, jerk is zero. If we go from 84 fps to a stop in 30 seconds:

$$\text{Acceleration} = \frac{0 \text{ fps} - 84 \text{ fps}}{30 \text{ seconds}} = -2.8 \frac{\text{feet}}{\text{second}^2}$$

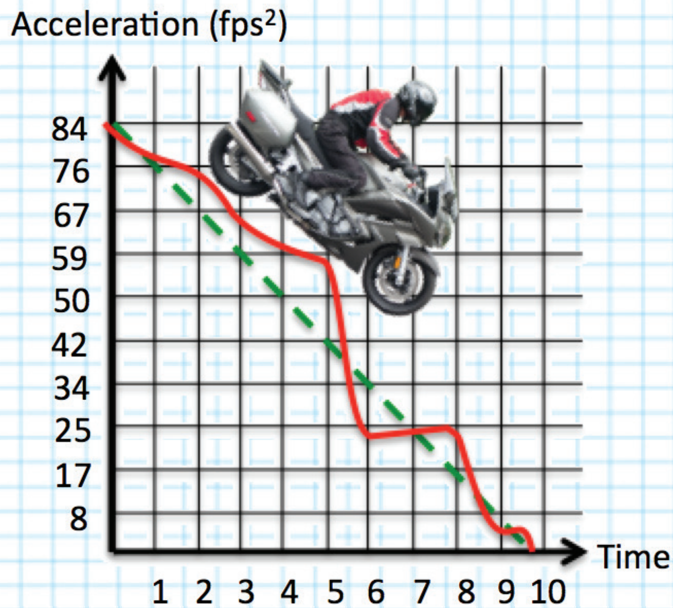
If we suddenly increase our deceleration to -5 feet/sec^2 in 5 seconds:

$$\text{Jerk} = \frac{(-2.8 - (-5))}{5 \text{ seconds}} = 0.44 \frac{\text{feet}}{\text{second}^3}$$

In other words: $\text{Jerk: } j = \frac{\Delta a}{t}$

Now let's say our intrepid rider underestimates the amount of braking required. If he or she then decides to really clamp down on the brakes, then backs off, and then on again, the stop will be said to be "jerky."

Why is jerk pertinent to a pilot? You can stop the airplane on a short runway using aggressive braking by applying your best guess of brake pedal and then holding the brakes steady. As the brakes heat up they become more effective and the deceleration will increase. But it will be done smoothly. Or you can apply some brake, a little less, a little more. Then the passengers will call you a jerk.



Gravity

Over the years we've come to realize that objects on earth fall at 32 ft/sec^2 which is 9.8 m/sec^2 for most of the world. When speaking of acceleration to earth — falling —, we substitute the term g for a so that:

Velocity under earth's gravity $v = gt$

And that means the distance resulting from earth's gravitational pull

$D = \frac{1}{2} gt^2$

Mass

[Dole, pg. 6]

Mass is a measure of the amount of material in a body. Weight, on the other hand, is a force caused by the gravitational attraction of the earth, moon, sun, or other heavenly bodies. Weight will vary, depending upon where the body is located in space. Mass will not vary with position.

Weight (W) = mass (m) X acceleration of gravity (g)

W = mg

Rearranging: $$m = \frac{w}{g} \quad \frac{lb}{ft/sec^2} = \frac{lb\text{-}sec2}{ft}$$

(Called a "slug")

Moments

[Dole, pg. 6]

Moments are measured by multiplying the amount of the applied force F by the moment arm l.

Moment: $M = Fl$

The moment arm is the perpendicular distance from the line of action of the applied force to the center of rotation. Moments are measured as foot pounds (ft-lbs) or as inch pounds (in.-lb).

Power

[Dole, pg. 9]

Power is defined as "the rate of doing work":

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}}$$

But distance / time = velocity, so: Power: Power = force X velocity

Friction

[Dole, pg. 10]

If two forces are in contact with each other, a force develops between them when an attempt is made to move them relative to each other. This force is called friction. Several factors are involved in determining friction effects on aircraft during takeoff and landing operations. Among these are runway surface material, condition of the runway, tire material and tread, the amount of brake slippage. All of these variables determine a coefficient of friction μ (mu). The actual braking force F_b is the product of this coefficient μ and the normal (squeezing) force between the tires and the runway:

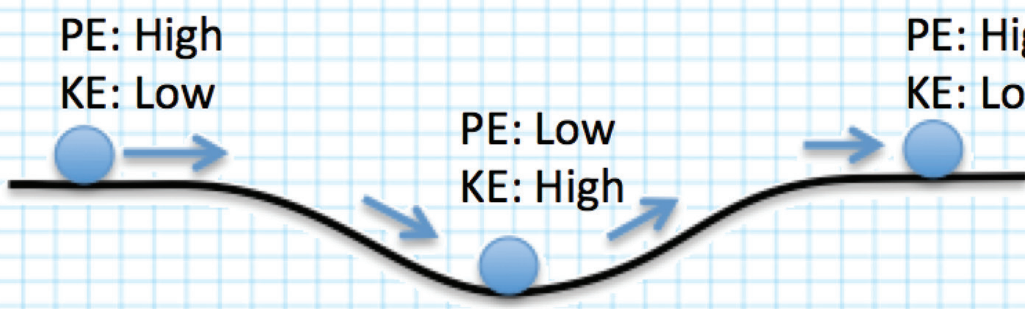
Braking Friction: $F_b = \mu N$

where F_b = braking force, μ = coefficient of friction, N = normal force on wheels.

Energy

Consider for a moment a metal ball traveling at a constant speed on a flat surface approaching a dip. We say the ball and the surface are both frictionless and there is no air resistance for the sake of this academic exercise. We know that as the ball goes down the dip in the surface, its speed will increase until it gets to the bottom where it will trade that extra speed to climb the ramp. When it ends up back at the original level it will again be at its original speed. Before it hits the slope, the energy of the ball can be described as the Kinetic Energy (KE) that comes from its motion and the Potential Energy (PE) due to its higher elevation.

Potential Energy: $PE = Wh$



[Dole, pg. 8]

Kinetic energy requires movement of an object. It is a function of the mass m of the object and its velocity v:

Kinetic Energy: $KE = \frac{1}{2}mv^2$

[Dole, pg. 8]

The total mechanical energy of an object is the sum of its PE and KE:

Total Energy: $TE = PE + KE$

[Dole, pg. 8]

The law of conservation of energy states total energy remains constant. Both potential and kinetic energy can change in value, but the total energy must remain the same; "Energy cannot be created or destroyed, but can change in form."